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Reflections on Russell Hardin

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"Führer befiehl, wir folgen dir!" Charismatic Leaders in Extremist Groups

Michael Baurmann, Gregor Betz, and Rainer Cramm

RUSSELL HARDIN'S ECONOMIC THEORY OF KNOWLEDGE

The economic approach to explaining individual behavior has undergone significant changes and enhancements in the last decades. Traditional rational choice is based on the presupposition of given preferences which, in the face of external restrictions and on the basis of subjective beliefs, are translated into action by rational decisions. The assumption that we can explain the behavior of people in general as a result of optimizing rational choices was contested already quite early by the theory of bounded rationality. Since then the overwhelming empirical findings of countless experimental and field studies have proved conclusively that people in their actual behavior practically never meet the rigorous requirements of standard rational choice theory.

The questioning of the presupposition of homogeneous and stable preferences does not go back so far as the attack on the assumption of rationality. But in the meanwhile, it is also part of a more or less mainstream

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criticism on rational choice theory to stress the empirical evidence we have for the heterogeneity of preferences, for example, in regard to altruistic and retributive preferences, the adaptation of aspiration levels to feasible opportunities, or the phenomenon that intrinsic motivation can be crowded out or reinforced due to contextual factors.

But one cornerstone of traditional rational choice had received amazingly little critical attention until Russell Hardin published his "How Do You Know" in 2009. The question how we can integrate an empirically convincing explanation of belief formation into a rational actor theory and sort out the role different kinds of beliefs play as motivating factors for human actions was not on the agenda of important research desiderata. This is somewhat astonishing as our beliefs about the facts in the world or the importance of certain values and norms are obviously decisive for our way of acting. Therefore the empirical processes by which we acquire these beliefs should have been of utmost interest for every theory of action.

In the case of normative beliefs, the neglect is maybe an even more serious omission, induced by the erroneous assumption that normative beliefs are just "cheap ideas" that have no real influence on human behavior. But, as Russell Hardin stresses straight at the outset of his book, we have to acknowledge that moral or religious principles come to many people as facts "no different in kind from other facts, such as the moon goes through its various phases" (Hardin 2009, 18). This kind of everyday objectivism does not only open up the possibility that people act according to their moral or religious beliefs just as regards their descriptive beliefs, but that they may adopt moral or religious beliefs that lead them to act in ways that are against their genuine interests (cf. 17)—a possibility that must be as irritating as it is fascinating for a rational actor theory.

Russell Hardin proposes an economic theory of knowledge as an approach to closing this gap in rational actor theory. The theory is economic in the sense that it strives to explain the knowledge base of average persons as being the result of choices in which people weigh up the costs and benefits of gaining certain pieces of knowledge (cf. 2ff.). Such a theory understands the acquisition of knowledge as an essentially rational process of considering the trade-offs between the value of any kind of knowledge and the value of other things which compete with the investment in knowledge acquisition: "The theory would not be about what the philosophical epistemologist's criteria for truth claims should be, but rather why we come to know what we know or believe" (xi). As the criteria ordinary persons apply to judging their beliefs "are not necessarily criteria for truth, but

merely and genuinely criteria of usefulness" much of the knowledge people accept and act on will be "merely satisficing knowledge, that is, good enough" (24f.). If we want to understand human behavior in this area what is required, therefore, is not a philosophically general theory of knowledge but a "street-level account" (Hardin 1992), a pragmatic theory that focuses on the actual ways people come to hold their beliefs but that bears little resemblance to the "theories of knowledge of those in ivory towers" (Hardin 2009, 19).

Implicated in this approach is a further deviation from philosophical epistemology by using a very broad concept of knowledge which follows the everyday use of this term, and makes no general distinction between beliefs and knowledge or between moral and factual knowledge. An economic theory of knowledge aims at including a vast area of various kinds of belief and behavior, "such as ordinary moral choice, religious belief and practice, political participation, liberalism, extremism, popular understandings of science, and cultural commitments" (3).

Russell Hardin's approach exhibits a family resemblance with social epistemology as it starts from the same basic and almost trivial fact that nearly all of our information and knowledge is not gained by our own experience, investigation, and deliberation but via testimony. Most of an individual's knowledge is socially generated and a result of a division of labor in the production of knowledge (cf. 5). We have no other option than to rely on others if we want to participate in the collective knowledge of our world. Both theories emphasize in this context the important role of epistemic authorities and experts. Contrary to scientific knowledge, ordinary knowledge "is almost entirely grounded in hearsay from a supposedly credible or even authoritative source" (1). So "we first have to judge a particular authority, and then we infer the truth of the authority's claim" (11). Hardin suggests that this deference to authority may be also essential in moral judgments as it "is only an extension of normal reasoning to let specialists assess religious matters and moral matters of right and wrong" (15).

But, in contrast to social epistemology, Russell Hardin is not interested in the question whether and under what conditions information via testimony could create "justified true beliefs". He is interested in the question how people in fact gain information and knowledge. An economic theory of knowledge is an empirical theory of epistemic processes, not a normative theory. However, as an economic theory of knowledge is an offspring of rational choice theory, it could at least be judged as weakly normative in that

it looks for a rational reconstruction of the factual processes of belief formation.

THE "CRIPPLED EPISTEMOLOGY" OF EXTREMISM

One field to which Russell Hardin applies an economic theory of knowledge is the phenomenon of extremist beliefs (cf. 185ff.). At first sight this may appear as a quite unusual subject for a theory of knowledge. In our paper we want to demonstrate the fruitfulness of this approach and—inspired by Russell Hardin's pioneer work—to describe and analyze a social-epistemic mechanism that can help to explain the emergence, stability, and erosion of extremist opinions in a group.

We thereby share two basic assumptions with Russell Hardin. First, that the acquisition of extremist beliefs follows the same patterns and processes as the acquisition of beliefs about the facts in the natural or social world. People come to believe the truth of extremist world views in the same way as they come to believe the truth of physics or the weather forecast. And we also agree that it is a "crucial move" for an explanation of extremist thinking when we recognize that people learn extremist ideas the same way they learn other things (cf. 159). Second, as the acquisition of most of our beliefs is to be explained as social and not as individual processes, this also applies to extremist beliefs. Hardin's general claim, already noted above, that in the course of these social processes people may adopt moral or religious beliefs that lead them to act in ways that are not in their interest is of special relevance when dealing with extremist or other deviant convictions.

Russell Hardin presents his approach as a serious alternative to psychological or traditional sociological explanations. He argues that we should analyze the dynamics of extremist thinking in groups as a social-epistemic process on the collective level and not as a process that can be attributed primarily to individuals and their idiosyncrasies: "It is generally the group that produces and sustains fanaticism" (185). That does not mean that Hardin abandons an individualistic methodology, but rather that we should understand the formation of individual convictions and opinions as a complex result of multifaceted interactions of people in their social networks and relations. Of course, how much variance such an epistemic approach actually could explain in this difficult and heterogeneous field is ultimately an empirical question.

If a social-epistemic process on the group level is crucial for the emergence and consolidation of extremist beliefs, it is essential to know the special characteristics of groups in which extremist thinking can flourish. Hardin focuses on three factors which he summarizes as "the crippled epistemology of extremism" (Hardin 2002). The first factor is the inflicted or self-chosen isolation of a group of like-minded people by which the beliefs of its members are constantly reaffirmed and may become more and more polarized. This can work even though for the overwhelming majority of other people outside the group these beliefs sound bizarre and absurd. The second factor is an effective norm of exclusion by which the less intensely committed members of a group and the moderates exit while the most dedicated and extremist remain. The third factor is the crucial role of epistemic authorities in propagating and transmitting extremist views in a group and the unconditional devotion of the group members to their ideological and political leaders.

Hardin summarizes the conditions for a crippled epistemology of groups: "If I am in a small community holding beliefs that others outside that community would think very odd, I may find those beliefs not at all odd because, after all, they are held by everyone I know. They may be merely part of the vast catalog of beliefs that I hold from dependence on authority" (Hardin 2009, 187).

CHARISMATIC LEADERS IN EXTREMIST GROUPS

The empirical evidence supports Hardin's analysis. Especially the impressive studies on religious fundamentalism of the "The Fundamentalism Project" (Chicago 1987–1995) which was directed by Martin E. Marty and R. Scott Appleby shows convincingly that groups can develop an idiosyncratic "enclave culture" which is successfully isolated from external influences and that the impact of "charismatic" leaders as ideological authorities is decisive in practically all groups for the inculcation and maintenance of fundamentalist world views.

The crucial role of charismatic leadership is especially salient for religious fundamentalism because the "holy texts" such as the Bible, the Thora, or the Koran reveal their alleged fundamentalist messages not without a heavily biased and selective interpretation. And in most religious traditions the interpretation of holy texts is the exclusive task of religious authorities who make the mission of these texts comprehensible for the ordinary believer and religious layperson. But as heretic religious groups do not recognize the official authorities of their institutionalized and "secularized" denominations, religious authority and leadership in these groups come into

being through an attribution of charismatic qualities to certain persons by the members of the group themselves. The ascription of extraordinary abilities, religious virtuosity, exceptional leadership and moral virtues is the basis for the enthronement of omnipotent religious and political authorities who are in a position to induce extremist and fundamentalist convictions among their followers (cf. Baurmann 2007, 2010a).

But we cannot be content with just stating the fact that the formation of certain variants of extremist groups is regularly dependent on the existence of charismatic leaders. The existence of a superior authority in a group is one possible explanatory factor; however, the emergence of such an authority *is in need of explanation itself.* Leadership does not operate in a vacuum but must be based on a group of potential followers who can be convinced and mobilized. The "charisma" of persons is therefore not a self-evident cause of their exceptional authority. It has to be clarified instead which social conditions and processes in a group lead to the attribution of a special "charisma" to certain persons so that they are established as supreme ideological leaders whose epistemic authority is so potent that they are able to generate devoted followers and convert them to radical believers that are normally rejected by the large majority of the surrounding society.

We can characterize this sovereign position of power as a position in which a person enjoys *exclusive epistemic trust* of the group members. This trust must be accompanied by a corresponding strict mistrust toward all people outside the group and toward competing epistemic authorities who on no account are to be accepted as alternative sources of information and knowledge. The emergence and consolidation of charismatic leadership in a group is necessarily combined with the formation of a group-specific *particularistic trust*—in the social as well as in the epistemic dimension.[1]

Epistemic trust includes social trust in the personal integrity and benevolence of persons and, in addition, confidence in their special competence and cognitive faculties which together can motivate others to accept and adopt their opinions and views. In the case of "charismatic" authorities, this can imply indoctrinating their followers with ideologies and convictions that differ significantly from their initial belief systems and world views: "it is written, but I tell you!" But even charismatic leaders cannot develop their messages in an empty space. They must connect with what is—already—*written* and present in the life world of their addressees. The more they manage to do this, the more plausible their message will appear and the less they have to utilize their "capital" of charisma to convince their followers. Therefore we have to take into account that the evolvement of radical and

extremist ideologies in a group will often be an incremental process in which the faith in certain leaders and the adoption of their views will develop mutually and gradually in a self-reinforcing dynamic.

Our central explanandum then is: *How can exclusive epistemic trust in a certain person evolve and stabilize in a group so that this person is able to implant and disseminate extremist and deviant views among the group members?*

A Social Mechanism of Opinion Dynamics

This process can be explained if we understand the underlying social mechanism. We assume that this mechanism is a special case of a social–doxactic mechanism which determines opinion dynamics in social groups in general (cf. Baurmann et al. 2014). The core of this mechanism is constituted by a process of mutual influence and adaptation in which individual experiences and deliberations are continuously compared and adjusted in accordance with the experiences and deliberations of other persons who are considered relevant and reliable. In detail we make the following assumptions:

1. Persons influence each other mutually in their opinions on the basis of *epistemic trust.* The greater the epistemic trust in a person, the more other people will orient themselves according to the opinions of this person.
2. Epistemic trustworthiness is based on *coherence, competence,* and *veracity. Coherence* means that the opinions of another person must appear plausible to be taken seriously, they should not diverge too much from one's own already established opinions but have to stay within a certain confidence interval or opinion space. *Competence* refers to the ability of a person to acquire reliable knowledge and sound insights in a certain area. *Veracity* is attributed if it is assumed that the incentives of the social context and the motivational dispositions of persons will lead them to transmit their knowledge and insights truthfully to their recipients.
3. *Epistemic self-confidence* is based on the competence persons ascribe to themselves. The lower the epistemic self-confidence of persons, the more they will be inclined to adapt to the opinions of other people who they judge to be epistemically trustworthy.
4. Opinion formation involves *first-order opinions* about the issues that are relevant in a certain field and *second-order opinions* about the

epistemic trustworthiness of persons who express their opinions about these issues. First-order opinions can include descriptive as well as normative opinions. Second-order opinions refer to characteristics of persons that are relevant for their quality as epistemic sources.

5. Persons *influence each other mutually* both in the formation of their first-order opinions and their second-order opinions. They consider the opinions of other trustworthy persons with regard to descriptive and normative issues as well as with regard to their estimation who is competent and reliable to pass considered judgments over these issues.

As noted above, these factors constitute a general socio-doxastic mechanism and as such do not signal any "abnormalities". Our central research hypothesis suggests that the emergence of extremist opinions in certain groups is the result of the nuts and bolts of this general mechanism and of the predominance of external conditions that constitute a deficient epistemic environment, much in the sense of Russell Hardin's crippled epistemology—meaning not as a result of psychology, irrationality, or individual deviance. To put it pointedly, one can become an extremist because one lives in a pathological epistemic environment and not because of a pathological personality (cf. Baurmann 2007).

It is an important feature of the described mechanism that it not only explains the group-induced development of first- and second-order opinions but that it also depicts the dynamic relationships between these different layers of opinion formation. On account of this structure, persons will be influenced by other persons not only in regard to their opinions about political options, societal connections, or ideological world views. This adaptation process itself will in turn be intertwined with the mutual adaptation of the second-order opinions about who has sufficient or special competence to understand and judge such options, connections, or world views. These two-layer dynamics could result in far-reaching transitions of the initial convictions of persons so that they ultimately may adopt extremist and radical opinions which were originally not within their opinion space and may well have appeared absurd to them.

We think that precisely in the interrelations between opinions of the first and second order lies the key to an explanation of how it can come about that even in a group in which initially neither an outstanding leader was generally accepted nor extremist views were held by the majority, a development can take place that finally leads to the establishment of an uncontested ideological leader under whose influence all other group members adopt convictions which differ drastically from their original world views.

But how such a mechanism works exactly and how the different factors affect its mode of action in detail are open questions. They are not to be answered easily, not least because the postulated mechanism exhibits a considerable internal complexity due to its multi-level structure. It is not possible to analytically determine the results of opinion dynamics in a group with many members after prolonged sequences of mutual influence on different levels or the impact smaller or larger changes of individual parameters or external conditions will produce. On the other hand, the basic elements of the supposed mechanism and their fundamental interrelations are quite simple. The challenges for analyses only begin when we have to deal with interrelations involving large numbers of actors over long periods.

Mechanisms of this kind, therefore, are predestined for experimental simulations. In the following we want to show how on the basis of an idealized mathematical model some of the fundamental aspects of the relevant dynamics could be explored with such simulations. These models and simulations could not themselves deliver explanations and they cannot substitute an empirical examination of theories. But they are potentially powerful instruments to develop new and fruitful hypotheses in a systematic and transparent way. They could help to illuminate the complexity of social dynamics and to detect concealed and analytically incomprehensible consequences of theoretical assumptions (cf. Hegselmann and Flache 1998).[2]

SIMULATION OF OPINION DYNAMICS IN EXTREMIST GROUPS

Structure of the Simulation Model

We have developed a simple prototype of a simulation model for opinion dynamics which provides promising first results (cf. Baurmann et al. 2014). The basic factors and relations which, according to our assumptions, are constitutive for the general social mechanism of opinion dynamics are operationalized in the model as follows[3]:

1. The model describes how the opinions of n agents change in the course of time (discrete time steps, $t = 0,1,2,3, ...$).
2. Each agent possesses a first-order opinion which is represented by a real number between 0 and 1.

3. Each agent assigns himself and the other agents degrees of epistemic competence on a scale between 0 and 1. Accordingly each agent possesses n second-order opinions.

4. An agent A trusts another agent B iff (i) B's first-order opinion are inside the confidence interval of A and (ii) A assigns according to his second-order opinions to B at least the same level of competence as to himself.[4]

5. *First dynamic principle*: the first-order opinions of an agent A at time step $t + 1$ equals the average of the first-order opinions of all agents at time step t whom A trusts at t.

6. *Second dynamic principle*: the second-order opinion of agent A about the degree of competence of B at time step $t + 1$ equals the average of the corresponding opinions of all agents at time step t whom A trusts at t.

Because the model abstracts from all other factors which influence our opinion formation as well (sympathy, argumentation, complexity, interests, emotions, etc.), it is a strongly simplified reconstruction of an in fact highly complex process. The model, therefore, is neither suitable for complete explanations nor prognostic aims (cf. Betz 2006, 2010). But, on the other hand, exactly because of its idealizations the model facilitates examination of the special aspects which are under consideration here with high precision and particularly rigorously. This will contribute to the heuristic value and explanatory potential of the hypotheses which are deducible from the model.

The outcomes will relate especially to the intertwined dynamics of first- and second-order opinions. With their help we can generate hypotheses about how it is possible that persons with extremist opinions can accumulate the necessary exclusive epistemic trust in a group to become a "charismatic" leader and in this way successfully disseminate extremist opinions that were initially outside the horizon of the other group members. Of course, whether such a process of mutual adaptation of first- and second-order beliefs in fact plays an important or maybe even decisive role in the emergence and dissemination of extremist world views can only be clarified by empirical studies.

Our model combines and extends the Lehrer-Wagner model (Lehrer and Wagner 1981) on the one hand and the Hegselmann-Krause-model (Hegselmann and Krause 2002, 2006; Hegselmann 2004) on the other hand. In both models beliefs are represented by real numbers in the unit interval. With the Lehrer-Wagner model, we share the idea that the involved persons mutually ascribe to each other different degrees of competence (second-order opinions).[5] But, as in the Hegselmann-Krause-model, the new beliefs of a person are not a result of just a weighted average but are subject to the bounded-confidence mechanism, respectively the coherence restriction. Particularly the inclusion of variable second-order opinions differentiates our approach from previous models.[6] This innovative element allows the reproduction and simulation of much more complex opinion dynamics than the alternative models.

We also think that interrelations between first- and second-order opinions are in fact an essential part of the empirically observable opinion formation processes. If this is the case, then simulation models should include this structure because these models should not only reproduce end states that are compatible with empirical facts but should also aim at reconstructing the causal mechanisms as adequately as possible (cf. Hedström and Swedberg 1998; Hedström and Ylikoski 2010).

A precise formal description of our model can be found in Appendix 1.

First Experiment: Emergence of Extremist Groups

As already stated, the ideological power of charismatic leaders is based on the exclusive epistemic trust of their followers which corresponds to a correlative mistrust toward all other epistemic sources and authorities. The findings of the "Fundamentalism Project" prove that all studied groups indeed make great efforts to secure particularistic in-group trust and social isolation and immunize their ideology against alternative world views and divergent experiences and influences. These strategies aim at ensuring that the group members will not develop any reliance on persons who do not belong to their own group.

Two simulation experiments with our model support the assumption that the absence or rather the undermining of external trust relations is just as crucial for the formation as for the stabilization of extremist groups and their internal hierarchical structure with a "charismatic" leader.

We analyze a group with 10 persons as members. Persons P2–P10 have moderate first-order opinions (0.5, 0.55, and 0.6); only person P1 takes an extreme position with a first-order opinion 0.9. The confidence interval of all persons is 0.33. The extreme position of P1 is compatible only with the confidence interval of P2 who holds the first-order opinion 0.6. The initial trust relations are depicted in Fig. 1a. Persons P5–P10 trust each other

mutually. The same applies to persons P2-P4. P2 and P3 in addition trust P5, and P2 also trusts P1. But P1 trusts nobody except himself. This maximal level of epistemic self-confidence is an important precondition for becoming a group-leader who is able to impose his personal opinions on the group. Because P1 does not concede to any other person the same degree of competence as himself, his self-confidence can neither be shaken by divergent second-order opinions of other persons nor will his extreme first-order opinions be challenged by more moderate views in his environment.

These assumptions model a situation in which a group (P1–P4) already experiences a significant degree of social isolation. Due to their thin episte-mic trust relations to persons outside their group their opinion formation is largely shielded against influences from outside. Therefore important pre-conditions for a "crippled epistemology" are fulfilled.

If we run a simulation of the opinion-formation process starting from this situation, already after a few steps a group evolves which is characterized by extreme opinions and an exclusive epistemic trust toward a charismatic leader who is the source of the progressive dissemination of these opinions in the group.

The first-order opinions of P2-P4 continuously adjust to the extreme position of P1 (cf. Fig. 1B). The initial trust relations of P2 and P3 with P5 (Fig. 1A, $t = 0$) are broken off step by step, and whereas P2 has trusted extremist P1 from the beginning, P3 and P4 follow him in steps 4 and 5 and also develop trust toward P1. P1 consequently becomes an uncontested authority (Fig. 1A, $t = 6$) who can impose his own extremist views without compromise on his new followers. It is noteworthy that by this process P3 and P4 accept an extremist position in the end, even though this position was outside their confidence interval at the beginning and must have appeared distinctly "implausible" to them because of the incompatibility with their already established beliefs. Decisive for the development of the extremist group is therefore the "intermediary" P2 who radicalizes the opinions of P3 and P4 at first only moderately until they finally enter the sphere of influence of P1.

Figure 1C demonstrates how second-order opinions play an essential role in these dynamics. It shows the development of the second-order opinions of P3. Up to the fifth step P3 judges P5, a member of the mainstream group, as at least as competent as herself. Consequently the trust relation to P5 stays intact and the subgroup of P3 is not yet completely isolated. But in the fifth step, P3 for the first time develops trust in the
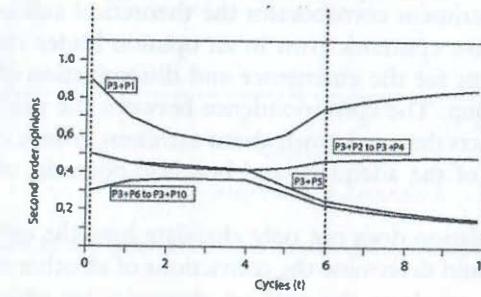

A: Trust networks at different times
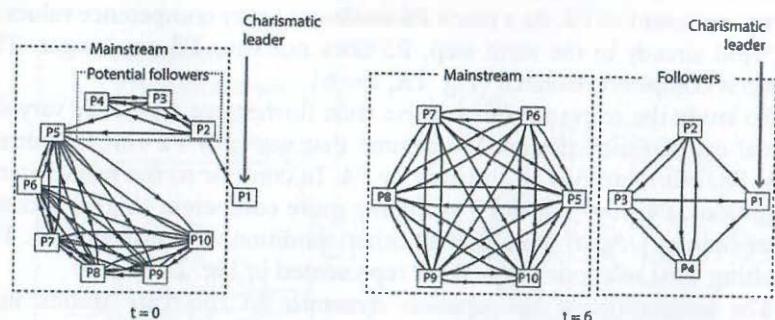
B: First-order opinions of group members
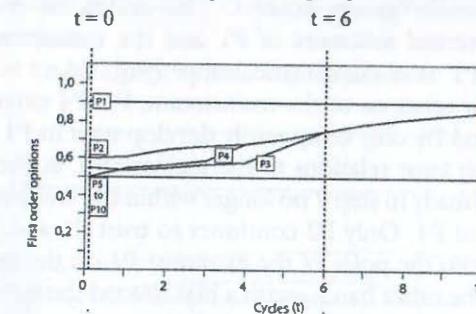
C: Second-order opinions of person P3

Numeric specification in Appendix 2.

Fig. 1 Simulation of the emergence of extremist groups, sufficient exclusivity of trust relations

potential leader P1 whose second-order opinions about P5 (0.1) therefore become relevant to P3. As a result P3 attributes lower competence values to P5. And already in the sixth step, P3 does not trust P5 any longer. The group is completely isolated (Fig. 1A, $t = 6$).

To study the relevance of exclusive trust further, we minimally vary the virtual experimental design. We assume that not only P2 and P3 initially trust P5, but that P5 is also trusted by P4. In contrast to the former initial conditions, P4 now judges P5 as slightly more competent (higher second-order opinion). Apart from that all other conditions remain identical. The resulting trust relations at $t = 0$ are represented in Fig. 2a.

The simulation of the opinion dynamics in this case results in a completely different picture although the starting conditions appear quite similar: no extremist group evolves. The additional trust relationship between the potential followers of P1 and the mainstream prevents the recognition of P1 as a charismatic leader (Fig. 2A, $t = 7$). Instead of breaking off their relations to the mainstream, P2–P4 extend them in fact. Moreover, P3 and P4 only temporarily develop trust in P1 (step 6). But as they deepen their trust relations to the mainstream, at the same time, the extremist P1 is already in step 7 no longer within the limits of the confidence interval of P3 and P4. Only P2 continues to trust P1 and positions herself eventually between the poles of the extremist P1 on the one hand and the mainstream on the other hand—with a bias toward the mainstream because P2 trusts more than one person there.

The first experiment corroborates the theoretical and empirical conjecture that exclusive epistemic trust in an opinion leader could be a crucial explanatory factor for the emergence and dissemination of extremist convictions in a group. The correspondence between the results of the experiment and the facts that are known about extremist groups could be deemed as an indicator of the adequacy and heuristic potential of the simulation model.

But the simulation does not only elucidate how the influence of charismatic leaders could determine the convictions of all other members of their groups. It also emulates the opinion dynamics by which in a stepwise transition of the trust relations in a group such a leading figure is established in the first place. This was the explanatory task we postulated: *By what social mechanism can exclusive epistemic trust in a certain person evolve in a group and establish that person as an uncontested epistemic authority?* In the simulation model such a mechanism is driven by the intricate interrelations between opinions of first and second order. The establishment of a
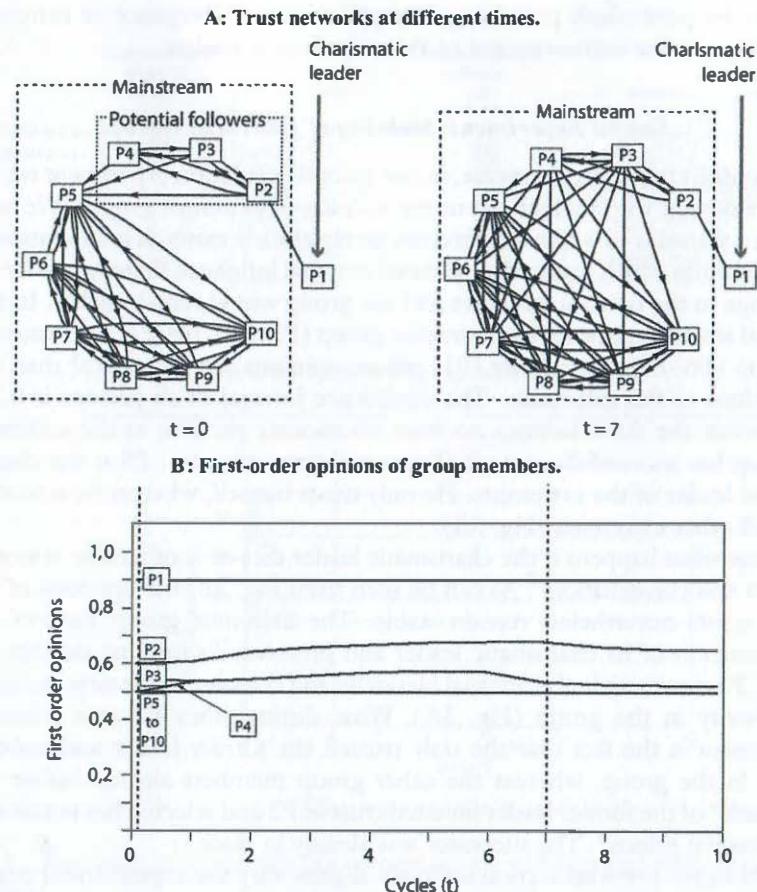
Fig. 2   Simulation of the emergence of extremist groups, insufficient exclusivity of trust relations

charismatic leader is the result of a mutual adaptation of the judgments of group members as to which persons are epistemically and socially trustworthy and which persons have to be regarded with suspicion. The results of the simulations, therefore, support the hypothesis that an explanatory approach that is based on the relationships between first- and second-order opinions

could be particularly promising in explaining the emergence of extremist groups and the enthronement of their ideological leaders.

Second Experiment: Stability of Extremist Groups

In contrast to the previous case, in our second simulation experiment we are not studying the emergence but the stability of extremist groups. We start with a situation in which an extremist group already exists. It is a situation in equilibrium which means that without external influence there would be no change in the opinion structure and the group would remain stable. In this initial situation, besides the extremist group (P2–P5), there is a mainstream group (P6–10) and a loner (P1) whose opinions are less radical than the opinions of the extremists. The confidence interval of all persons is 0.25. Between the three factions no trust whatsoever prevails, as the extremist group has successfully cut off all external trust relations. P5 is the charismatic leader of the extremists. He only trusts himself, whereas he is trusted by all other extremists (Fig. 3A).

But what happens if the charismatic leader dies or is otherwise removed from this constellation?[7] As can be seen from Fig. 3B, the opinions of the extremists nevertheless remain stable. The extremist group survives the elimination of its charismatic leader and preserves its internal stability. In fact, P2 moves up in the internal hierarchy and constitutes the new exclusive authority in the group (Fig. 3A). What distinguishes P2 as a potential successor is the fact that she only trusted the former leader and nobody else in the group, whereas the other group members already before the "death" of the former leader invested trust in P2 and selected her in this way as "crown prince". The successor was already in place.

As in the previous section we again slightly vary the experimental design to explore variations in the significance of external trust relations. In this experiment the members of the extremist group P3 and P4 do not only trust the other extremists P2 and P5, but also trust the "loner" P1 (Fig. 4A, $t = 0$). Consequently, the opinions of P3 und P4 (0.8) in the initial equilibrium are positioned between the opinions of the charismatic leader (0.9) and the "loner" (0.6). This constellation is also endogenously stable, but in this case the extremist group dissolves as soon as the charismatic leader P5 is removed (Fig. 4B). Without P5 the balance between extremist and external authorities is changed from the point of view of P3 and P4. After the disappearance of their highly trusted leader, P3 and P4 at first tent towards the "loner" P1. But in adjusting their first-order opinions to the

A: Trust networks at different times

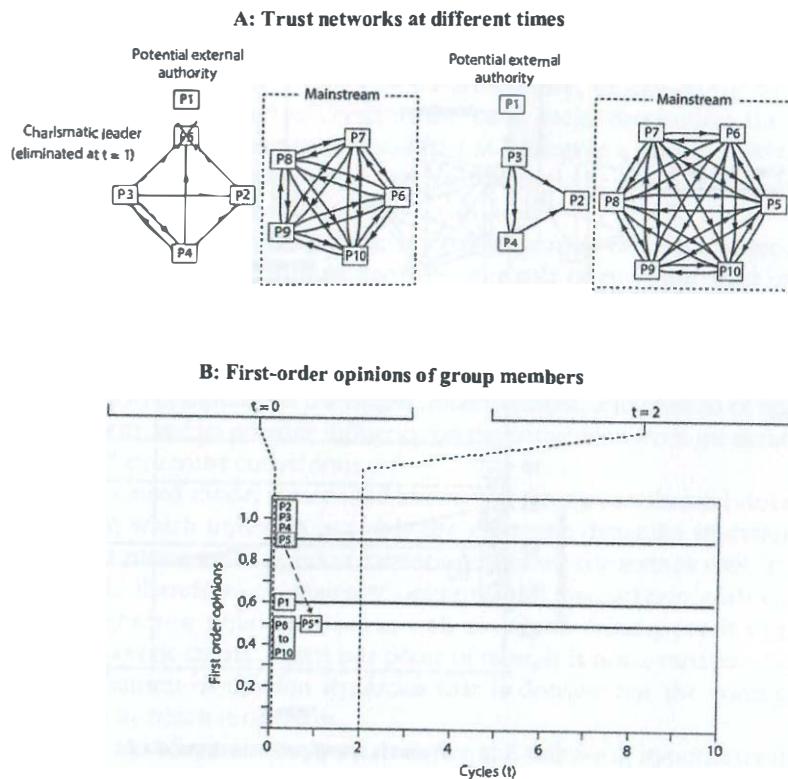B: First-order opinions of group members

Numeric specification in Appendix 2.

Fig. 3 Simulation of the stability of extremist groups, sufficient exclusivity of trust relations

opinions of P1, eventually also the opinions of the mainstream inhabitants were included in their confidence interval. As the trusted persons with moderate opinions greatly outnumber the trusted persons with extremist opinions, P3 and P4 depart more and more from the remaining extremist until they do not trust her at all and are integrated fully in the mainstream.

Because the initially stable extremist group has failed to cut all external trust relations, it collapses when the charismatic leader is eliminated. This outcome of the simulation suggests that not only for the emergence but also for the maintenance of extremist groups it is crucial that they establish and
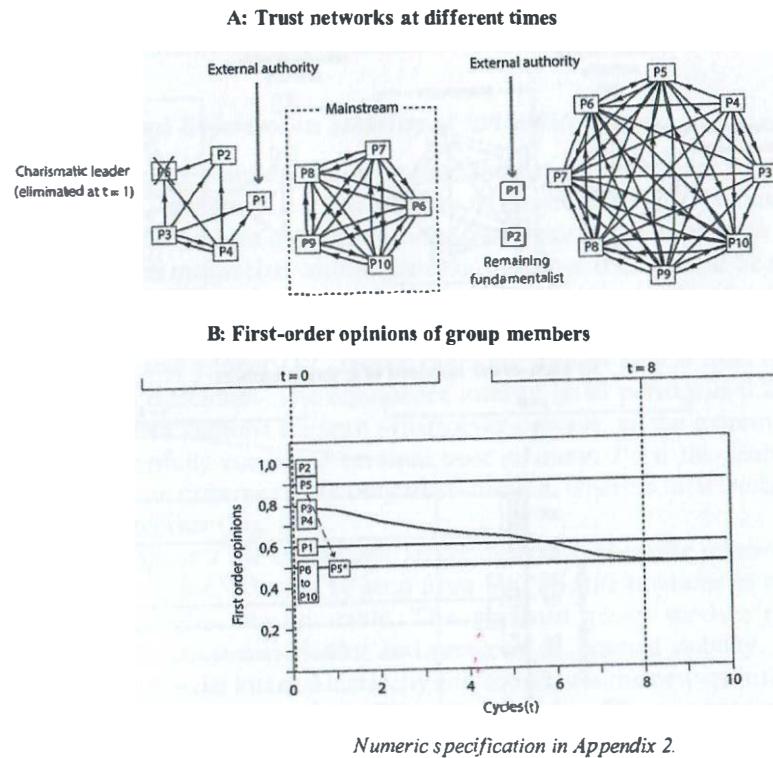
A: Trust networks at different times

B: First-order opinions of group members

Numeric specification in Appendix 2.

Fig. 4 Simulation of the stability of extremist groups, insufficient exclusivity of trust relations

preserve particularistic in-group trust. In the long run, only such extremist groups can survive which successfully prevent external trust relations of their members and are ready to undertake serious efforts in providing resources to secure epistemic seclusion, social isolation and their "crippled epistemology".

HYPOTHESES

The first results of our simulation model demonstrate that even with this simple prototype informative and interesting hypotheses about the conditions for the emergence and continued existence of extremist groups can be generated. The simulations support and reproduce the empirical finding that charismatic leaders can play an essential role in the dissemination and stabilization of extremist world views. Furthermore, we can on the basis of this model simulate and understand the basic social mechanism through which certain persons are first established as leaders in a group. Lastly, the instability and the erosion of extremist groups could be explained as a result of opinion dynamics under modified conditions.

The core of the modeling is the mutual adaptation of first- and second-order beliefs, or, to put it more generally: the role of epistemic trust in the formation of beliefs. Only if one systematically considers beliefs which refer to concrete spheres of life as well as beliefs which deal with epistemic competence and trustworthiness can one accomplish a sufficient level of complexity to comprehend the origin, establishment, and erosion of epistemic authority and its possible influence on the conversion from moderate to radical and extremist convictions.

The proposed model is intended as a model for a general social-doxastic mechanism which underlies not only the epistemic dynamics in extremist groups but processes of opinion formation in other contexts as well. It can be applied, therefore, to majority opinions and mainstream convictions about religious or political issues as well as fashion trends, youth subcultures, or esoteric circles. From our point of view, it is not a variation of the basic mechanism of opinion dynamics that is decisive but the contextual conditions in which it operates.

In the case of extremism, we can derive the following hypotheses from our experimental simulations:

1. Trust in a potential ideological leader must not initially be especially strong or exclusive. Existing trust relations toward moderate persons could be eroded in the process of opinion formation. Not all members of an extremist group must therefore be social outcasts from the start.
2. Charismatic leaders can come from outside with only weak trust relations to members of a group at the outset. It can be sufficient for them to become a group leader if only single members of the group trust them. This allows for promising infiltration strategies which are targeted only at a few people.
3. Unshakable self-confidence combined with a general disregard for the competence of other persons is a crucial precondition to become a charismatic leader. Persons with lower self-confidence will tend to subordinate themselves more and more to such leader personalities.

4. Extremist opinions can gradually become plausible and must not be inside the opinion space of the majority of group members from the beginning. There can be a self-reinforcing process of radicalization which takes place stepwise and sequentially.
5. Weak trust relations with the mainstream can immunize a group against extremist opinions. Relatively small shifts in these relations can tip a development and a critical threshold can easily be exceeded. Therefore it is an important strategy of extremist groups to combat this hazard potential and sever their member's external trust relations by all means.
6. Weak trust relations with outsiders can undermine extremist opinions in a group. Persons who are not part of the mainstream but do not express a radical position can build bridges for reintegration of extremists into the mainstream.

These hypotheses can be put in a nutshell: *taking the opinions of others seriously can be sufficient to become an extremist!*

As already emphasized, simulation models are highly idealized reproductions of reality which cannot substitute empirical validation of theories and deliver explanations per se. However, the simulation experiments with our prototype elucidate that such models can have a significant heuristic value and are suitable to analyze the basic mechanisms of complex social dynamics and to generate fruitful hypotheses. In our case the results are an additional support for Russell Hardin's ingenious theory of the "crippled epistemology" of extremist groups, and we recommend it as an excellent framework for future research in this troubling field.

APPENDIX 1: THE MODEL

The model describes the collective opinion formation in a group of n persons ($G = \{1, \ldots, n\}$) in discrete time steps ($t = 0, 1, \ldots$). Each person $i \in G$ at a given point of time t has precisely one first-order opinion, $x_i(t) \in$ (with $j = 1, \ldots, n$), and n second-order opinions, $y_{[i,j]}(t) \in [0.1]$ (with $j = 1, \ldots, n$). The set $V_i(t)$ of all group members who are trusted by person i at time t is defined as:

$$V_i(t) := \{j \in G : (|x_i(t) - x_j(t)| \leq \epsilon) \wedge (y_{i,j}(t) \geq y_{i,i}(t))\}. \quad (1)$$

whereby $\epsilon \in \mathbb{R}$ is a confidence parameter. The first- and second-order opinions of a person i are modified according to the following dynamic rules

$$x_i(t+1) = \frac{1}{|V_i(t)|} \sum_{k \in V_i(t)} x_k(t) \quad (2)$$

$$y_{i,j}(t+1) = \frac{1}{|V_i(t)|} \sum_{k \in V_i(t)} y_{k,j}(t). \quad (3)$$

APPENDIX 2: NUMERIC SPECIFICATION OF THE SIMULATION EXPERIMENTS

A simulation experiment is numerically completely specified by

- the group size n,
- the confidence interval ϵ,
- the initial first-order opinions

$$X(0) = \begin{pmatrix} x_1(0) \\ \vdots \\ x_n(0) \end{pmatrix},$$

the initial second order opinions

$$Y(0) = \begin{pmatrix} y_{1,1}(0) & \cdots & y_{1,n}(0) \\ \vdots & \ddots & \vdots \\ y_{n,1}(0) & \cdots & y_{n,n}(0) \end{pmatrix}.$$

Emergence of extremist groups (Fig. 1)
- Group size: $n = 10$,
- Confidence interval: $\epsilon = 0.33$,
- Initial first- and second-order opinions:

$$\mathbf{X}(0) = \begin{pmatrix} 0.9 \\ 0.6 \\ 0.55 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix},$$

$$\mathbf{Y}(0) = \begin{pmatrix} 0.9 & 0.5 & 0.5 & 0.5 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.9 & 0.5 & 0.5 & 0.5 & 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.9 & 0.5 & 0.5 & 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.9 & 0.5 & 0.5 & 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}.$$

Emergence of extremist groups (Fig. 2)

- Group size: $n = 10$,
- Confidence interval: $\varepsilon = 0.33$,
- Initial first- and second-order opinions:

$$\mathbf{X}(0) = \begin{pmatrix} 0.9 \\ 0.6 \\ 0.55 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix},$$

$$\mathbf{Y}(0) = \begin{pmatrix} 0.9 & 0.5 & 0.5 & 0.5 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.9 & 0.5 & 0.5 & 0.5 & 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.9 & 0.5 & 0.5 & 0.5 & 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.9 & 0.5 & 0.5 & 0.5 & 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}.$$

Stability of extremist groups, endogenous stable situation with charismatic leader (Fig. 3, $t = 0$)

- Group size: $n = 10$,
- Confidence interval: $\varepsilon = 0.25$,
- Initial first- and second-order opinions:

$$\mathbf{X}(0) = \begin{pmatrix} 0.6 \\ 0.9 \\ 0.9 \\ 0.9 \\ 0.9 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix},$$

$$\mathbf{Y}(0) = \begin{pmatrix} 0.7 & 0.05 & 0.6 & 0.6 & 0.6 & 0.65 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.3 & 0.8 & 0.45 & 0.45 & 0.9 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.3 & 0.55 & 0.5 & 0.5 & 0.8 & 0.35 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.3 & 0.55 & 0.5 & 0.5 & 0.8 & 0.35 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.3 & 0.8 & 0.45 & 0.45 & 0.9 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}.$$

Stability of extremist groups, without charismatic leader (Fig. 3, $t > 0$)

- Group size: $n = 10$,
- Confidence interval: $\varepsilon = 0.25$,
- Initial first- and second-order opinions:

$$\mathbf{X}(0) = \begin{pmatrix} 0.6 \\ 0.9 \\ 0.9 \\ 0.9 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix},$$

$$\mathbf{Y}(0) = \begin{pmatrix} 0.7 & 0.05 & 0.6 & 0.6 & 0.6 & 0.65 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.3 & 0.8 & 0.45 & 0.45 & 0.3 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.3 & 0.55 & 0.5 & 0.5 & 0.4 & 0.35 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.3 & 0.55 & 0.5 & 0.5 & 0.4 & 0.35 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}.$$

Stability of extremist groups, endogenous stable situation with charismatic leader (Fig. 4, $t = 0$)

- Group size: $n = 10$,
- Confidence interval: $\varepsilon = 0.25$,
- Initial first- and second-order opinions:

$$\mathbf{X}(0) = \begin{pmatrix} 0.6 \\ 0.9 \\ 0.8 \\ 0.8 \\ 0.9 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix},$$

$$\mathbf{Y}(0) = \begin{pmatrix} 0.7 & 0.05 & 0.6 & 0.6 & 0.6 & 0.65 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.7 & 0.8 & 0.45 & 0.45 & 0.9 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.7 & 0.55 & 0.5 & 0.5 & 0.8 & 0.35 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.7 & 0.55 & 0.5 & 0.5 & 0.8 & 0.35 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.7 & 0.8 & 0.45 & 0.45 & 0.9 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}.$$

Stability of extremist groups, without charismatic leader (Fig. 4, $t > 0$)

- Group size: $n = 10$
- Confidence interval: $\varepsilon = 0.25$
- Initial first- and second-order opinions:

$$\mathbf{X}(0) = \begin{pmatrix} 0.6 \\ 0.9 \\ 0.8 \\ 0.8 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{pmatrix},$$

$$\mathbf{Y}(0) = \begin{pmatrix} 0.7 & 0.05 & 0.6 & 0.6 & 0.6 & 0.65 & 0.6 & 0.6 & 0.6 & 0.6 \\ 0.7 & 0.8 & 0.45 & 0.45 & 0.3 & 0.2 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.7 & 0.55 & 0.5 & 0.5 & 0.4 & 0.35 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.7 & 0.55 & 0.5 & 0.5 & 0.4 & 0.35 & 0.4 & 0.4 & 0.4 & 0.4 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}.$$

NOTES

1. Russell Hardin does not like the term "trust" in this context because he wants to reserve the use of this term for relations with "strong" ties (Hardin 2009, 26). Insofar we use a thin concept of trust which also includes relations which are impersonal but share important aspects with personal trust relations such as dependence or risk-taking. However, this is a terminological, not a substantial point of departure (cf. Baurmann 2010b).
2. The explanatory significance of such modeling is discussed in a special issue of *Erkenntnis* (vol. 70, no. 1, January 2009) "Economic Models as Credible Worlds or as Isolating Tools?" with contributions among others by Nancy Cartwright, Till Grüne-Yanoff, Tarja Knuuttila and Robert Sugden.
3. For an application of this model to a "veritistic" issue cf. Betz et al. (2013).
4. In this prototype, we do not differentiate between the attribution of competence and veracity but subsume both under "competence".

5. But in contrast to the Lehrer-Wagner model, the competence degrees are not used in our model as weights for averaging but only to select trustworthy persons.
6. Deffuant et al. (2002) and Deffuant (2006) refine the bounded-confidence model to study the dynamics of polarization and radicalization processes but they do not consider second-order opinions. The same applies to a recent publication by Hegselmann and Krause (2015) in which they explicitly deal with the dissemination of extremist beliefs but without including the formation of epistemic trust relations.
7. Technically the charismatic leader P5 is not removed from the simulation but becomes part of the mainstream.

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